Written Reexam at the Department of Economics summer 2021

Economics of the Environment and Climate Change

Final Reexam

August 19, 2021

(3-hour closed book exam)

Answers only in English.

This exam question consists of 5 pages in total, including this front page.

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WRITTEN REEXAM IN
THE ECONOMICS OF THE ENVIRONMENT AND CLIMATE CHANGE
Spring 2021

Question 1. Climate policy in a Solow-type growth model (Indicative weight: ¾)

We consider an economy where output is produced by means of three factors of production: labour and two types of capital. Since the labour force is assumed to be constant, we do not include it explicitly in the production function; it is just present behind the scene. The two types of capital are called "black" capital and "green" capital, respectively. Black capital is plant and equipment that needs to use fossil fuel to be productive, so the use of this type of capital generates CO_2 emissions. Green capital may be thought of as solar panels, wind turbines and any other forms of capital that are operated by using renewable energy which does not cause CO_2 emissions. Thus, by substituting green for black capital one can reduce emissions per unit of output. The model uses the following notation:

Y = total output (GDP)
A = total factor productivity
B = black capital
G = green capital
K = total capital stock
E = emissions of CO_2
S = concentration of CO_2 in the atmosphere
s = savings rate (exogenous)
g = rate of technological progress (exogenous)
t = time (treated as a continuous variable)
$\dot{x} \equiv dx/dt$ = derivative of variable x with respect to time

Without loss of generality, we can choose our units of measurement such that the use of one unit of black capital generates an emission of one tonne of CO_2 each period. The model then contains the following equations, where $\alpha, \beta, \gamma, \delta, g, s$, and A_0 are all constant parameters:

Output:	$Y_t = A_t B_t^{\alpha} G_t^{\beta}, \qquad 0 < \alpha < 1, \quad 0 < \beta < 1,$		(1)
Total factor productivity:	$A_t = A_0 e^{gt} S_t^{-\gamma}, \quad A_0 > 0, \quad g > 0, \quad \gamma > 0,$		(2)
Total capital stock:	$K_t = B_t + G_t,$		(3)
Emissions of CO_2:	$E_t = B_t,$		(4)
Evolution of capital stock:	$\dot{K}_t = sY_t, \qquad 0 < s < 1,$		(5)
Evolution of CO_2 concentration:	$\dot{S}_t = E_t - \delta S_t, \qquad \delta > 0.$		(6)

Question 1.1: Discuss briefly how one can motivate the presence of the term $S_t^{-\gamma}$ in Equation (2). Also, discuss briefly why Equation (6) includes the term $-\delta S_t$.

Now suppose for a moment that the resource allocation in the economy is fully controlled by a social planner who wants to stabilize the concentration of CO_2 in the atmosphere at a constant level $\bar{S}$ so as to keep global warming at a tolerable level.

Question 1.2: Derive the constant level of emissions, $\bar{E}$, and the constant stock of black capital, $\bar{B}$, which will ensure that the CO_2 concentration is kept constant at the desired level $\bar{S}$. Show that when the desired CO_2 concentration has been achieved, total output may be written as

$$Y_t = \bar{A} e^{gt} \bar{B}^{\alpha} G_t^{\beta}, \qquad \bar{A} \equiv A_0 \bar{S}^{-\gamma}. \tag{7}$$

Note: The purpose of the next three questions is to guide you towards a solution for the economy's steady-state growth rate which will be useful for the subsequent analysis of climate policy.

Question 1.3: Define a new variable $y_t \equiv Y_t / G_t$ and use (7) to show that

$$\frac{\dot{y}_t}{y_t} = g - (1 - \beta) \frac{\dot{G}_t}{G_t}. \tag{8}$$

Then show that when S_t is kept constant, we must have $\frac{\dot{G}_t}{G_t} = s y_t$ so that

$$\frac{\dot{y}_t}{y_t} = g - (1 - \beta) s y_t. \tag{9}$$

Question 1.4: Explain why (9) implies that y_t will stabilize at the constant level

$$\bar{y} = \frac{g}{s(1-\beta)}. \tag{10}$$

Question 1.5: Now define the growth rate of output, $g_t^Y \equiv \dot{Y}_t/Y_t$, and use (7) plus your previous result $\frac{\dot{G}_t}{G_t} = sy_t$ and (10) to show that in the long run when S_t has been stabilized at $\bar{S}$, the growth rate of output will stabilize at the constant steady-state level

$$g^Y = \frac{g}{1-\beta}. \tag{11}$$

Give an intuitive economic explanation for the impact of the parameter β on the steady-state growth rate.

We now assume that resource allocation in the economy described by equations (1) through (6) is in fact governed by market mechanisms, but that the government can impose a carbon tax at the rate τ_t per tonne of CO_2 emitted, and that the government can also grant a subsidy to green investors at the rate σ_t per unit of green capital installed. In other words, the owners of black capital must pay a total carbon tax bill equal to $\tau_t E_t = \tau_t B_t$ per period, while the owners of green capital receive a total subsidy amount equal to $\sigma_t G_t$ per period. The government's net revenue is returned to the private sector as a lump sum transfer (or its net revenue need is covered by a lump sum tax) that does not affect investment decisions.

Question 1.6: Capital owners are free to invest in the type of capital that yields the highest marginal return net of taxes and subsidies. Explain (by using (1)) that a capital market equilibrium therefore requires that

$$\alpha \frac{Y_t}{B_t} - \tau_t = \beta \frac{Y_t}{G_t} + \sigma_t. \tag{12}$$

What is the magnitude of the Marginal Cost of Abatement (MAC), that is, the marginal social cost of reducing CO_2 emissions by one tonne?

Question 1.7: Now suppose the government does not levy a carbon tax (i.e., $\tau_t = 0$) but only uses the subsidy σ_t to achieve its climate policy target. Suppose further that the subsidy rate σ_t is continuously adjusted so that CO_2 emissions are kept at the constant level derived in Question 1.2 ensuring that the CO_2 concentration is kept constant at the level $\bar{S}$ (implying that $B_t = \bar{B}$). From our previous analysis we know that the economy will then converge on a steady state where (10) and (11) hold. Now use (12) with $\tau_t = 0$ to derive an expression for the ratio of the government's total subsidy bill to output, $\sigma_t G_t/Y_t$. How will this ratio evolve over time? Will the government be able to maintain the subsidy policy in the long run?

Question 1.8: Suppose instead that the government does not offer a subsidy to green capital ($\sigma_t = 0$) but only imposes a carbon tax ($\tau_t > 0$) which is continuously adjusted to ensure that $E_t = \bar{E}$, implying that the stock of black capital is kept at the level $\bar{B}$ which stabilizes the CO_2 concentration at $\bar{S}$ so that, once again, the economy converges on the steady state (10) and (11). Now use (12) to derive an expression for the ratio of total carbon tax revenue to GDP, $\tau_t \bar{B}/Y_t$. How will this ratio evolve over time? Will the government be able to maintain its carbon tax policy in the long run? Is the tax policy preferable to the subsidy policy, or vice versa? Motivate your answer.

Question 2. Green growth (Indicative weight: 1/4)

Discuss whether "green growth" is possible? (*Note: This question may be answered without any use of math and/or graphical analysis. However, you are welcome to use math or diagrams to the extent that you find it convenient*).